

Make Slow Fast - how to speed up interacting disordered matter

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Anderson and dynamical localization have been experimentally observed with ultra-cold atomic matter. Feshbach resonances are used to efficiently control the strength of interactions between atoms. This allows to study the delocalization effect of interactions for localized wave packets. The delocalization processes are subdiffusive and slow, thereby limiting the quantitative experimental and numerical analysis. We propose an elegant solution of the problem by proper ramping the interaction strength in time. We demonstrate that subdiffusion is speeded up to normal diffusion for interacting disordered and kicked atomic systems. The door is open to test these theoretical results experimentally, and to attack similar computational quests in higher space dimensions

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Introduction

The quantum wave nature of ultracold atoms in optical potentials [1], as demonstrated impressively through their macroscopic condensation [2, 3], is the key ingredient for the recent observation of Anderson localization with quantum atomic matter [4, 5]. Quasi-one-dimensional elongated traps are modulated randomly with speckle potentials [6, 7], or simply quasiperiodically with interfering laser beams [8], in order to observe the halt of spreading of an initially localized wave packet of 10^4 - 10^5 Rb and K atoms, and an exponentially localized atomic density distribution profile. The length scales are controlled by the localization length ξ which is a function of the potential parameters, and possibly also the energies of packet atoms. This phenomenon of wave localization is inherently relying on the phase coherence of matter waves. It is closely related to the dynamical localization of the quantum kicked rotor in momentum space, which was successfully probed already in 1995 using ultra-cold Na atoms [9]. Recent experiments with quasiperiodically kicked rotors with Cs atoms extend to two- and three-dimensional disorder potentials [10]. Interestingly systems of one- and two-dimensional optical waveguides have been also recently used to probe Anderson localization [11, 12].

For some atomic species (K, Cs, Na, Li) Feshbach resonances can be used to efficiently control the strength of interactions between atoms [13–16]. This opens the possibility to study the fate of Anderson localization for interacting localized wave packets. Indeed the first experiment of this kind [17] showed that interaction beats localization, but in a very slow way - the second moment m_2 of an atomic wave packet increases subdiffusively in time: $m_2 \sim t^\alpha$ with $\alpha < 1$. This process may stop in the long run once the atomic density n of the wavepacket reaches the inverse of the localization length ξ (which touches the quantum world of many-body localization [18]). On shorter times (when typically more than 10 atoms occupy one local single particle state) the mean field approximation is a reasonable tool for the study of the subdiffusive process.

The problem

The mean-field approximation replaces the many body linear Schrödinger equation in a hugely dimensional Hilbert space with a nonlinear Schrödinger equation (NLS), e.g. the Gross-Pitaevsky equation. The effective interaction strength β is proportional to the scattering length a_s . What matters in terms of quality here, is the fact that for almost any disorder (or quasiperiodic) potential realization the corresponding NLS will be nonintegrable. This seemingly unimportant mathematical property has a very profound impact - the dynamics of a wave packet becomes in general chaotic in time, characterized by positive Lyapunov coefficients and exponential divergence of nearby trajectories. As a consequence the coherence of phases of waves which constitute a given initial wave packet is lost, and with it also the whole effect of wave localization. First observed in 1993 by Shepelyansky for an NLS version of the quantum kicked rotor [19], it was recently studied with great detail for random and quasiperiodic potentials [20–24]. The main outcome for quasi-one-dimensional models with two-body atomic interactions is an asymptotic wavepacket spreading with $\alpha = 1/3$ [25]. With inverse time units equal to single particle kinetic energies the crossover from intermediate to universal asymptotic dynamics takes place at dimensionless time $\tau \sim 10^6$. In the Florence setup [17] largest times reached are $\tau \sim 10^4$, leaving the experiments in the intermediate case-to-case-dependent dynamics. Although the onset of subdiffusion is clearly observed, no reliable experimental data are currently at hand to measure the exponent

α , as follows from the data analysis and the large statistical errors in [17]. While some experimental optimization and increase of the kinetic energy may add one order of magnitude in time, another one-two orders are needed and are probably currently out of range of accessibility. Notably similar problems of insufficient available time scales arise with computational studies when turning to higher dimensional analogs [23, 26, 27]. While two-dimensional models appear to be at the edge of reasonable analysis, three-dimensional are clearly not. As follows from the above we can diagnose the problem of lacking time scales for a safe observation and study of subdiffusive interacting atomic cloud dynamics in disordered media.

The solution

Instead of trying to substantially increase available time scales, we propose here to speed up the subdiffusive process itself. This is done by a temporal ramping of the two-body interaction strength, which can be varied e.g. for K atoms by three orders of magnitude close to the Feshbach resonance [13]. Why should that help? The momentary diffusion rate D of a spreading packet in one spatial dimension is proportional to the fourth power of the product of interaction strength β and particle density n : $D \sim (\beta n)^4$ [28]. In the course of cloud spreading the density n decreases, and therefore also D . This is the reason for the predicted subdiffusion process, which is substantially slower than normal diffusion. We propose here to compensate the decrease of the density n with an increase in the interaction strength β . Depending on the concrete time dependence $\beta(\tau)$ we expect different faster subdiffusion processes, and possibly even normal diffusion. The condition for that outcome to be realized is, that the internal chaos time scales (basically the inverse Lyapunov coefficients) will be still short enough so that the atomic cloud can first get chaotic, and then spread. With that achieved, the cloud spreading will be faster, and we can expect that the available experimental time will suffice for the precise observation and analysis of the process.

Let us get into numbers for one spatial dimension. The second moment is $m_2 \sim 1/n^2$ and the momentary diffusion constant $D \sim (\beta n)^4$. For a constant β the solution of $m_2 = D\tau$ yields $m_2 \sim 1/n^2 \sim \tau^{1/3}$, and therefore $n \sim \tau^{-1/6}$. Thus we choose now a time dependence $\beta \sim \tau^\nu$. Then the resulting spreading is characterized by

$$m_2 \sim \tau^{(1+4\nu)/3}, \quad d = 1. \quad (1)$$

For $\nu = 1/2$ we already obtain normal diffusion $m_2 \sim \tau$.

Similar for two spatial dimensions, where $m_2 \sim 1/n$, for a constant β the cloud spreading is even slower with $m_2 \sim \tau^{1/5}$. With a time dependent ramping $\beta \sim \tau^\nu$ the resulting speedup is

$$m_2 \sim \tau^{(1+4\nu)/5}, \quad d = 2. \quad (2)$$

For $\nu = 1$ we again obtain normal diffusion.

Once ramping is too fast, we expect to see several different scenarios. Either fragmenting atomic clouds appear since some parts of the cloud get self-trapped [29, 30] and some other parts do not. If self-trapping is avoided, we may also see ramping-induced diffusion: while the internal cloud dynamics does not suffice to decohere phases, initial fluctuations in the density distribution can lead to considerably different temporal energy renormalizations in different cloud spots, and therefore to an effective dephasing similar to a random noise process in real time and space.

Results in one dimension

Here we study the spreading of atomic clouds in one-dimensional disorder potentials and in a quantum kicked rotor with interacting atoms. The first model is described with the discrete NLS (DNLS)

$$i \frac{\partial \psi_l}{\partial \tau} = \epsilon_l \psi_l + \beta(\tau) |\psi_l|^2 \psi_l - \psi_{l+1} - \psi_{l-1} \quad (3)$$

in which the on-site energy ϵ_l is chosen uniformly from a $[-W/2, W/2]$ random distribution. The nonlinear quantum kicked rotor (NQKR) is studied within the diagonal interaction approximation introduced by Shepelyansky in [19]:

$$\psi_l(\tau + 1) = \sum_m (-i)^{l-m} J_{l-m}(k) \psi_m(\tau) e^{-i \frac{\bar{\tau}}{2} m^2 + i \beta(\tau) |\psi_m|^2}, \quad (4)$$

where $\psi_l(\tau)$ are the Fourier coefficients of the corresponding time-dependent many body wave function. $J_{l-m}(k)$ is a Bessel function of the first kind, whose argument k is the kick strength, and $\bar{\tau}$ is a parameter which relates the period of applied kicks T (set to $T = 1$) to the natural frequency of rotor, defined as $\omega = \hbar/2M$ (M -mass of atoms). In both

models β is the interaction strength ramped in time τ - the dimensionless time for the DNLS and the number of kicks for the NQKR model:

$$\beta(\tau) = \begin{cases} \beta_0 & , \tau \leq \tau_0 \\ \beta_0 \left(\frac{\tau}{\tau_0}\right)^\nu & , \tau > \tau_0 \end{cases} \quad (5)$$

In both models we consider a wave packet which is initially concentrated on a single site for purely technical reasons, without any loss of generality (extended initial clouds are perfectly usable as well [31]). After some first time scale τ_0 the packet spreads approximately over one localization length ξ . The total norm of the packet is set to one without any loss of generality and is proportional to the total number of atoms in a cloud (similarly, we could also choose any larger norm and rescale β accordingly). To characterize the spreading of the cloud we compute the density $n_l = |\psi_l(\tau)|^2$, the participation number $P = 1/\sum_l n_l^2$ (the number of strongly excited sites), and the second moment $m_2 = \sum_l (l - \bar{l})^2 n_l$ (the squared distance between the wave packet tails), where $\bar{l} = \sum_l l n_l$ is the first moment. In the NQKR the average energy of the atomic cloud is proportional to the corresponding second moment, $E = \frac{1}{2}m_2$. Another remarkable difference between both systems is that for large values of β self-trapping can occur for atomic clouds in disordered spatial systems which may lead to soliton formation [32]. For the kicked rotor case this is impossible since the interaction strength β action in (4) is cyclic reflecting the circumstance of periodic kick action in momentum space [33].

Equation (3) was time evolved using a SABA-class symplectic integration scheme and equation (4) as an iteration map. The parameters were fixed to $\beta_0 = 1$ and $W = 4$ for the DNLS and $\beta_0 = 0.4$, $\bar{\tau} = 1$ and $k = 3$ for the one-dimensional NQKR. Different realizations for the DNLS were produced by choosing different unique random sequences in the interval $[-W/2, W/2]$, while for the NQKR they were realized by exciting different initial states.

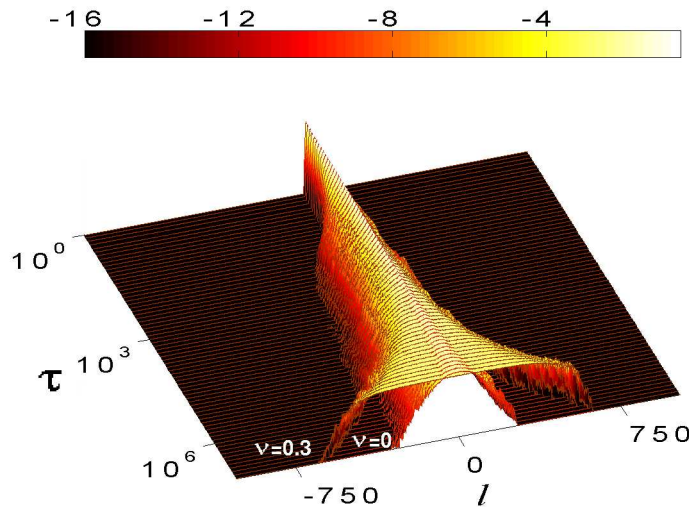


FIG. 1: Evolution of the averaged norm density $\langle n_l(\tau) \rangle$ in the case without ($\nu = 0$) and with ramping ($\nu = 0.3$) in log scale for the DNLS model.

The spreading of wave packets in the DNLS model, without and with ramping of the nonlinearity are shown in Fig. (1). Clearly packets spread faster when the nonlinearity is ramped in time. To quantify the spreading exponent, we averaged the logs (base 10) of P and m_2 over 1000 different realizations and smoothed additionally with locally weighted regression [34]. The (time-dependent) spreading exponents are obtained through central finite difference method [35], $\alpha = \frac{d\langle \log_{10}(m_2) \rangle}{d(\log_{10}(\tau))}$. The results for the DNLS and NQKR model are shown in Fig. (2). The exponents of subdiffusive spreading reach the theoretically predicted values. Moreover, for faster ramping of nonlinearity, the asymptotic state with constant exponent is reached faster. Monitoring of the participation number P for the DNLS indicates that self-trapping starts to occur already for $\nu = 0.4$. Results for the NQKR model, in which the self-trapping is avoided, confirm the reaching of a normal diffusion process for $\nu = 0.5$. Remarkably, the absence of self-trapping for the NQKR results in superdiffusion on intermediate times for $\nu > 0.5$. Finally, the exponent relaxes back to the normal diffusion value indicating the realization of ramping-induced diffusion. This case is illustrated for $\nu = 1.5$ in Fig. (2).

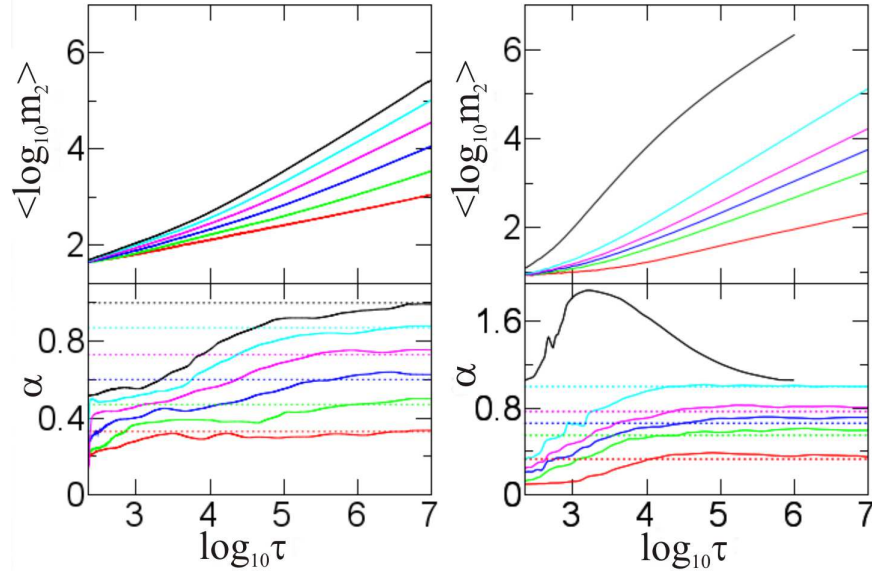


FIG. 2: *Left column:* the second moments (upper) and their power-law exponents α (lower) for the DNLS model for $\nu = 0$ (red), $\nu = 0.1$ (green), $\nu = 0.2$ (blue), $\nu = 0.3$ (magenta), $\nu = 0.4$ (cyan), and $\nu = 0.5$ (black). *Right column:* the second moments (upper) and their power-law exponents α (lower) for the NQKR model for $\nu = 0$ (red), $\nu = 0.17$ (green), $\nu = 0.25$ (blue), $\nu = 0.33$ (magenta), $\nu = 0.5$ (cyan), and $\nu = 1.5$ (black). Dashed colored lines correspond to expected values for exponents in both cases.

Results in two dimensions

To speed up subdiffusive processes in higher dimensions we considered the two-dimensional NQKR model based on the map introduced in [36], with an additional phase term which takes into account interactions in the diagonal approximation:

$$\psi_{l_1, l_2}(\tau + 1) = \sum_{s_1, s_2} (-i)^{s_1 + s_2} J_{s_1}(k/2) J_{s_2}(k/2) \psi_{l_1 - s_1 - s_2, l_2 + s_1 - s_2}(\tau) e^{-i \frac{\tau}{2} ((l_1 - s_1 - s_2)^2 + (l_2 + s_1 - s_2)^2) + i \beta(\tau) |\psi_{l_1 - s_1 - s_2, l_2 + s_1 - s_2}(\tau)|^2}. \quad (6)$$

The notation is the same as in the one-dimensional case, except that now we have two indices, for each possible direction. Note that according to the relation for Bessel functions $J_{-n}(x) = (-1)^n J_n(x)$, the wave packet ψ_{l_1, l_2} , defined by expression (6), exhibits symmetry with respect to l_1 and l_2 direction. The density is defined as $n_{l_1, l_2} = |\psi_{l_1, l_2}(\tau)|^2$, the participation number as $P = 1 / \sum_{l_1, l_2} n_{l_1, l_2}^2$, and the second moment is $m_2 = \sum_{l_1, l_2} [(l_1 - \bar{l}_1)^2 + (l_2 - \bar{l}_2)^2] n_{l_1, l_2}$, where $\bar{l}_1 = \sum_{l_1, l_2} l_1 n_{l_1, l_2}$ and $\bar{l}_2 = \sum_{l_1, l_2} l_2 n_{l_1, l_2}$. Equation (6) was time evolved as an iteration map for the fixed parameters $\beta_0 = 0.4$, $\bar{\tau} = 1$ and $k = 2$.

In Fig. (3) we compare the wave packet evolution for $\nu = 1$ (normal diffusion) and $\nu = 0$ at three different moments of time. We clearly observe the symmetry of wave packet and a much more violent spreading in the presence of ramping. The spreading exponents are computed similar to the one-dimensional case. We find very good agreement with the theoretical prediction (Fig. (4)). Again the asymptotic spreading state is reached faster for stronger ramping.

Conclusion

We have investigated the speeding up of the subdiffusive spreading in interacting disordered and kicked atomic systems by a proper ramping of the interaction strength in time. We confirm that ramping the interaction strength leads to faster subdiffusion. For fast enough ramping we even reach normal diffusion of atomic clouds. Self-trapping effects in disordered systems are limiting further speed up of the wave packet spreading. Most importantly the concept works equally well in one-dimensional and two-dimensional systems.

Our results on how to speed up slow subdiffusive processes in interacting disordered matter will be useful for quantitative experimental and computational studies of the impact of interactions on disorder induced matter wave

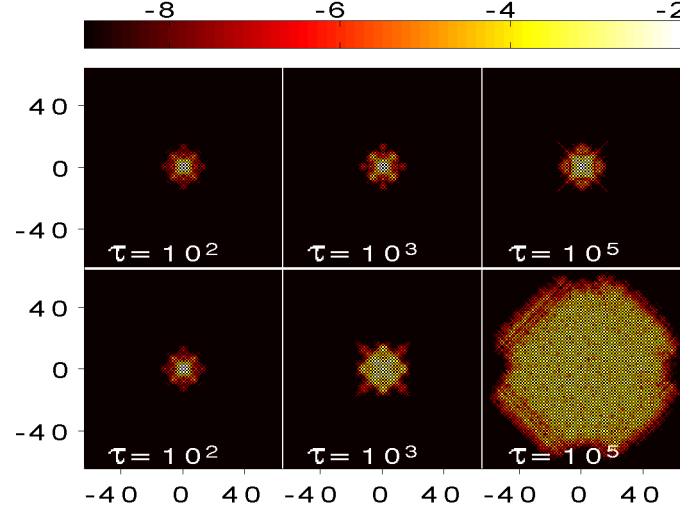


FIG. 3: The norm density $n_{l_1, l_2}(\tau)$ in the case without ($\nu = 0$) (upper row) and with ramping ($\nu = 1.0$) (lower row) after $\tau = 10^2$, $\tau = 10^3$, and $\tau = 10^5$ kicks in log scale for the two-dimensional NQKR model.

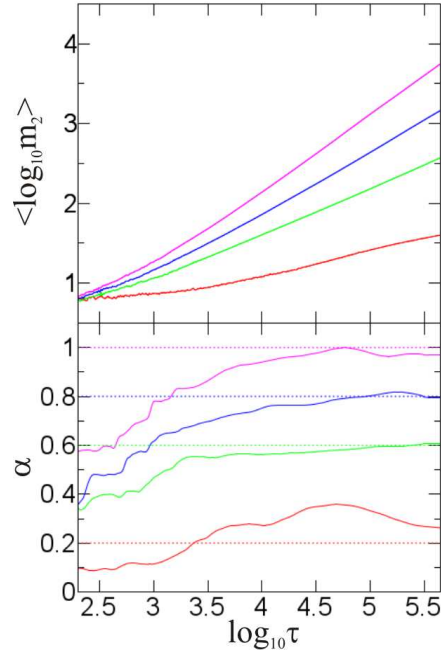


FIG. 4: The second moments (upper) and their power-law exponents α (lower) for the two-dimensional NQKR model for $\nu = 0$ (red), $\nu = 0.5$ (green), $\nu = 0.75$ (blue), and $\nu = 1$ (magenta). Dashed colored lines correspond to expected values for exponents.

localization. This is particularly true for experimental realizations with ultra-cold atoms, where the scattering length and thus the interactions strength can be tuned via Feshbach resonances by magnetic field variations. On the other hand, our results are also very useful for computational studies of spreading regimes in higher dimensional systems, where even modern computers reach their limits before reaching the subdiffusive asymptotics.

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